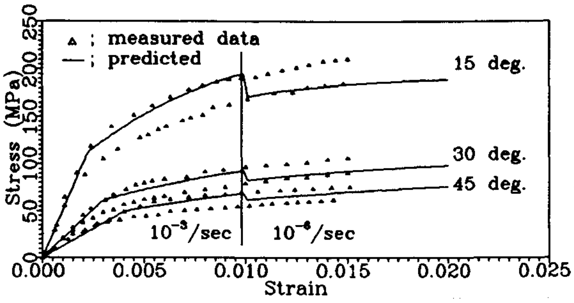

Fig. 3 Measured and predicted stress-strain curves of off-axis specimens subjected to a rapid change (decrease) in strain rate at 250°F.

Conclusions

The role of matrix in viscoplastic behavior of thermoplastic composites is identified in this study. On the basis of the fact that thermoplastic composite is a mixture of isotropic materials (fiber and matrix), the viscoplastic behaviors of thermoplastic composites with various off-axis angle are simulated by using the "unmixing-mixing" concept and a modified Bodner-Partom flow rule as a matrix viscoplastic constitutive model. As a consequence, this work gives us the simplicity in handling of the elastic anisotropy of the thermoplastic composites and makes it possible that the viscoplastic behavior of thermoplastic composites with any off-axis angle can be predicted by the viscoplastic properties of the matrix only. The concept of this study can be extended to the analysis of the laminated structures by solving boundary-value problems.

References

[1]Dvorak, G. J., and Bahei-El-Din, Y. A., "Plasticity Analysis of Fibrous Composites," *ASME Journal of Applied Mechanics,* Vol. 49, June 1982, pp. 327-335.

[2]Yoon, K. J., "Characterization of Elastic-Plastic And Viscoplastic Behavior of AS4/PEEK Thermoplastic Composite," Ph.D. Dissertation, School of Aeronautics and Astronautics, Purdue Univ., West Lafayette, IN, 1990.

[3]Gates, T. S., and Sun, C. T., "Elastic/Viscoplastic Constitutive Model for Fiber Reinforced Thermoplastic Composites," *AIAA Journal,* Vol. 29, No. 3, 1991, pp. 457-463.

[4]Tsai, S. W., and Hahn, H. T., *Introduction to Composites Materials,* Technomic Publishing Co., Lancaster, PA, 1980, pp. 388-401.

[5]Bodner, S. R., and Partom, Y. "Constitutive Equations for Elastic-Viscoplastic Strain-Hardening Materials," *ASME Journal of Applied Mechanics,* Vol. 42, No. 2, 1975, pp. 385-389.

[6]Kim, S. J., and Oden, J. T., "Finite Element Analysis of a Class of Problems in Finite Elastoplasticity Based on the Thermodynamical Theory of Materials of Type N," *Computer Methods in Applied Mechanics and Engineering,* Vol. 53, 1985, pp. 277-302.

[7]Kim, S. J., and Oden, J. T., "Generalized Flow Potentials in Finite Elastoplasticity—II. Examples," *International Journal of Engineering Science,* Vol. 23, No. 5, 1985, pp. 515-530.

Substructure Decomposition Method for the Control Design of Large Flexible Structures

M. Sunar* and S. S. Rao†
Purdue University, West Lafayette, Indiana 47907

Introduction

BECAUSE of the increasing demands of high structural performance requirements, the control of large flexible structures has attracted a considerable amount of research in recent years. There are two basic steps involved in the control of a flexible structure using discrete methods. In the first step, the structure is modeled, usually by the finite element method (FEM) that, for a large flexible structure, results in a large number of ordinary differential equations. These dynamic equations of motion (EOM) have to be used in the second step to determine a control law appropriate for the structure. In many cases, it is not practical or feasible to consider the full dynamic model of the structure for the global controller design. The decentralized control design is very attractive in the active control of large flexible structures, since it permits the design of controllers at component level and avoids many complications that may otherwise be present in the controller design for the whole structure. Recently, an approach called the controlled component synthesis was developed by Young[1] in which an interlocking control concept was used to minimize the motion of the nodes adjacent to the boundaries of the substructures.

In this work, a general decentralized control approach is presented for large flexible structures. The structure is decomposed into substructures for which the linear quadratic regulator (LQR) theory is used to determine the decentralized controllers at substructure level (subcontrollers). The reaction forces at the substructure boundaries were balanced by the control forces generated within the substructures. The subcontroller of each substructure is then used to assemble the controller for the whole structure. The computational effort required in the approach is substantially less compared with the complete structural model.

Substructure Decomposition

It is assumed that the flexible structure is decomposed into r substructures, where the kth substructure has neighboring s substructures. Note that in the subsequent discussions the subscript k refers to the kth substructure. Let $\{I\}_k$ denote the set of internal degrees of freedom (DOF) of the kth substructure, $\{B_1\}_k$ represent the set of boundary DOF between the kth and $(k+1)$th substructures, and so on. The EOM of the kth substructure can be represented as

$$M_k\ddot{\boldsymbol{x}}_k + C_k\dot{\boldsymbol{x}}_k + K_k\boldsymbol{x}_k = D_k\boldsymbol{u}_k \tag{1}$$

where $\boldsymbol{x}_k$ and $\boldsymbol{u}_k$ denote the vectors of displacement and control input, respectively; and M_k, C_k, K_k, and D_k denote the mass, damping, stiffness, and input weighting matrices in the configuration space, respectively. The matrices M_k, C_k, K_k, and D_k have to be summed properly to preserve the displacement compatibility of the whole structure.

The EOM of the kth substructure, Eq. (1), can be rearranged by grouping together the internal and boundary DOF of the substructure. The internal and boundary DOF of a substructure will vary depending on the surrounding substructure considered. Considering the kth substructure and its neighboring $(k+1)$th substructure, the set of internal DOF is given by $\{A_k\} = (\{I_k\}, \{B_2\}, \ldots, \{B_s\})_k$ and the set of boundary DOF by $\{B_k\} = \{B_1\}_k$. Hence the partitioned EOM for the kth substructure can be stated as

$$\begin{bmatrix} M_{AA} & M_{AB} \\ M_{BA} & M_{BB} \end{bmatrix}_k \begin{Bmatrix} \ddot{x}_A \\ \ddot{x}_B \end{Bmatrix}_k + \begin{bmatrix} C_{AA} & C_{AB} \\ C_{BA} & C_{BB} \end{bmatrix}_k \begin{Bmatrix} \dot{x}_A \\ \dot{x}_B \end{Bmatrix}_k + \begin{bmatrix} K_{AA} & K_{AB} \\ K_{BA} & K_{BB} \end{bmatrix}_k \begin{Bmatrix} x_A \\ x_B \end{Bmatrix}_k = \begin{bmatrix} D_{AA} & D_{AB} \\ D_{BA} & D_{BB} \end{bmatrix}_k \begin{Bmatrix} u_A \\ u_B \end{Bmatrix}_k \tag{2}$$

By using the Guyan static condensation method, the following relation can be written:

$$\begin{Bmatrix} x_A \\ x_B \end{Bmatrix}_k = \begin{bmatrix} -K_{AA}^{-1}K_{AB} \\ I_B \end{bmatrix}_k x_{Bk} = T_k x_{Bk} \tag{3}$$

Received June 6, 1991; revision received Jan. 28, 1992; accepted for publication Feb. 4, 1992.

*Graduate Student, School of Mechanical Engineering.
†Professor, School of Mechanical Engineering.

where T_k is the transformation matrix for the kth substructure while considering the interaction with the $(k+1)$th substructure. By defining proper sets for $\{A_k\}$ and $\{B_k\}$, the transformation matrices between the kth substructure and the other substructures can be determined using Eq. (3).

Controller Design and Balance of Interaction Forces for Substructures

The EOM of the kth substructure in state-space form are given by

$$\dot{z}_k = A_k z_k + F'_{Bk} z_k + B_k u_k = A'_k z_k + B_k u_k \tag{4}$$

where A_k and B_k are the system and state-space input matrices, respectively; u_k is the vector of control input generated within the kth substructure; and z_k is the state vector for kth substructure defined as

$$z_k = \{x_k \; \dot{x}_k\}^T = \{x_{Ak} \; x_B \; \dot{x}_{Ak} \; \dot{x}_{Bk}\}^T \tag{5}$$

and also

$$F'_{Bk} z_k = \sum_{i=1}^{s} \begin{Bmatrix} 0 \\ M_k^{-1} F_{Bik} \end{Bmatrix} \tag{6}$$

where F_{Bik} is the actuator (controller) force generated by the ith substructure surrounding the kth substructure. Note that F_{Bik} denotes the actuator force passed from the previous iteration. The input vector u_k in Eq. (4) is found using the LQR method as

$$u_k = -G_k z_k = -[R^{-1} B^T P]_k z_k \tag{7}$$

where P_k satisfies the following algebraic Riccati equation:

$$P_k A'_k + A'^T_k P_k - P_k B_k R_k^{-1} B_k^T P_k + Q_k = 0 \tag{8}$$

Recall that the input weighting matrix of the kth substructure is defined as

$$B_k = \begin{bmatrix} 0 \\ M_k^{-1} D_k \end{bmatrix} \tag{9}$$

where D_k is the input weighting matrix of the kth substructure in the configuration space. Postmultiplying Eq. (9) by Eq. (7) yields

$$B_k u_k = \begin{bmatrix} 0 \\ -M_k^{-1} D_k G_k \end{bmatrix} z_k \tag{10}$$

In the current iteration, the actuator force that is generated within the kth substructure to balance the interacting forces from the surrounding substructures (and to suppress vibration) can be determined using Eq. (10) as

$$f_k = \begin{Bmatrix} f_{Ak} \\ f_{Bk} \end{Bmatrix} = [-M_k M_k^{-1} D_k G_k] z_k = [-D_k G_k] \begin{Bmatrix} x_{Ak} \\ x_{Bk} \\ \dot{x}_{Ak} \\ \dot{x}_{Bk} \end{Bmatrix} \tag{11}$$

where f_{Ak} and f_{Bk} are the actuator forces at the internal and boundary DOF of the kth substructure, respectively. By defining proper sets of $\{A\}_k$ and $\{B\}_k$ for the kth substructure and each of its surrounding substructures, Eq. (3) can be used to condense the actuator force, f_k in Eq. (11), into the boundary DOF of the surrounding substructures. Consequently, Eq. (11) can be expressed as

$$f_k = \begin{Bmatrix} f_{Ak} \\ f_{Bk} \end{Bmatrix} = [-D_k G_k T'_k] \begin{Bmatrix} x_{Bk} \\ \dot{x}_{Bk} \end{Bmatrix} \tag{12}$$

where

$$T'_k = \begin{bmatrix} T_k & 0 \\ 0 & T_k \end{bmatrix} \tag{13}$$

Note that f_{Bki}, which is the actuator force generated by the kth substructure and condensed into the boundary DOF of the ith surrounding substructure, is not the same as F_{Bki} $(-F_{Bik})$ since they are obtained at two different iterations. At the final iteration, however, their magnitudes are the same.

If the kth substructure has no actuator(s), then the EOM of all its surrounding substructures must be modified. This can be done by lumping the mass, damping, and stiffness matrices of the kth substructure into those of the surrounding substructures. If the ith substructure is connected to the kth substructure, its EOM are modified as follows:

$$\begin{bmatrix} M_{AAi} & M_{ABi} \\ M_{BAi} & M_{BBi} + M'_k \end{bmatrix} \begin{Bmatrix} \ddot{x}_{Ai} \\ \ddot{x}_{Bi} \end{Bmatrix} + \begin{bmatrix} C_{AAi} & C_{ABi} \\ C_{BAi} & C_{BBi} + C'_k \end{bmatrix} \begin{Bmatrix} \dot{x}_{Ai} \\ \dot{x}_{Bi} \end{Bmatrix} + \begin{bmatrix} K_{AAi} & K_{ABi} \\ K_{BAi} & K_{BBi} + K'_k \end{bmatrix} \begin{Bmatrix} x_{Ai} \\ x_{Bi} \end{Bmatrix} = \begin{bmatrix} D_{AAi} & D_{ABi} \\ D_{BAi} & D_{BBi} \end{bmatrix} \begin{Bmatrix} u_{Ai} \\ u_{Bi} \end{Bmatrix} \tag{14}$$

where

$$M'_k = T_k^T M_k T_k, \qquad C'_k = T_k^T C_k T_k, \qquad K'_k = T_k^T K_k T_k \tag{15}$$

and T_k is found from Eq. (3) as discussed before.

The previous procedure is iterated until the closed-loop eigenvalues of the whole structure converge. Convergence does not mean, however, the convergence to the closed-loop eigenvalues of the complete model of the structure in the exact sense, for the approximation always exists due to the Guyan condensation scheme. Convergence is rather due to the better representation of the control forces at the substructure boundaries as the iterations progress, since the iterations start with the assumption that no actuator is present in the first substructure. Finally, the global controller for the whole structure is obtained by assembling the substructural gain matrices into the global gain matrix. The technique is illustrated by the following numerical example.

Numerical Example

The 18-bar planar truss shown in Fig. 1 is a cantilevered square truss with dimensions as indicated in the figure. The truss is made of aluminum with $E = 10^7$ psi and $\rho = 0.1$ lb/in.^{3}. The cross-sectional areas of the members of the truss are $A_i = 1$ in.^{2}, $i = 1, \ldots, 18$. The truss has 24 DOF in the state space. The actuators that can generate forces in the x and y directions are placed at nodes 2, 3, 5, and 6. The state and input weighting matrices Q and R are selected as $10^5 * I_Q$ and I_R where I_Q and I_R are the identity matrices with sizes 24×24 and 8×8, respectively. The structure is decomposed into four substructures as shown in Fig. 1. The displacement response of DOF 2 of the structure to a unit impulsive force applied at

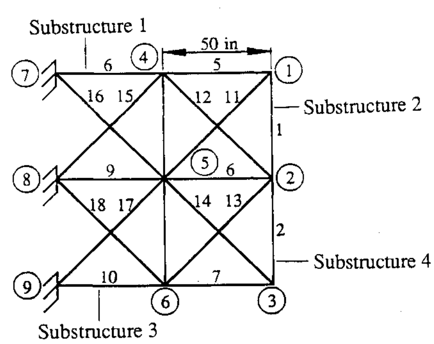

Fig. 1 Eighteen-bar truss with four substructures.

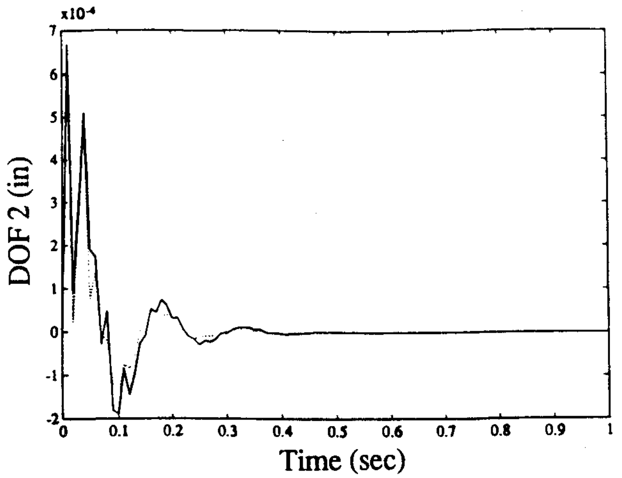

Fig. 2 Displacement of 18-bar truss at DOF 2 (*y* direction at node 1) in response to a unit impulse at DOF 2: ———, complete structural model; · · ·, substructure model.

DOF 2 using both the complete structural model and the substructures method is shown in Fig. 2. It can be seen in Fig. 2 that the responses given by the substructures method closely match those given by the complete structural model. This proves that the substructure decomposition method yields very accurate closed-loop system characteristics of the complete structure with a substantially smaller computational effort.

Conclusion

A substructure decomposition methodology has been developed to design the global controllers of large flexible structures from the decentralized subcontrollers designed at the substructure level. After partitioning the whole structure into substructures, transformation matrices between the substructures have been found by using the static condensation method. After determining the size of actuators for substructures, the control forces have been transformed into the boundary DOF of concerned substructures using the proper transformation matrices. A numerical example has been considered to show the efficiency of the method.

Reference

[1]Young, K. D., "Distributed Finite-Element Modeling and Control Approach for Large Flexible Structures," *Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 4, 1990, pp. 703–713.